Received: *October 18, 2000*, Accepted: *October 26, 2000*

Some exact results on tachyon condensation in string field theory

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Abstract: The study of open string tachyon condensation in string field theory can be drastically simplified by making an appropriate choice of coordinates on the space of string fields. We show that a very natural coordinate system is suggested by the connection between the worldsheet renormalization group and spacetime physics. In this system only one field, the tachyon, condenses while all other fields have vanishing expectation values. These coordinates are also well-suited to the study of D-branes as solitons. We use them to show that the tension of the D25-brane is cancelled by tachyon condensation and compute exactly the profiles and tensions of lower dimensional D-branes.

Keywords: Bosonic Strings, D-branes.

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Contents

1. Introduction

The problem of open string tachyon condensation on unstable branes in bosonic and supersymmetric string theory is interesting, since it touches on important issues in string theory such as background independence, off-shell physics, the symmetry structure of the theory, and the role of closed strings. In the context of string field theory (SFT) the main approach to this problem has been through Witten's cubic, or Chern-Simons string field theory [1], and in the past year notable progress has been made (see e.g. [2, 3]).

On the other hand, the physics of tachyon condensation is well understood from the first quantized (worldsheet) point of view. The endpoint of condensation is a state in which the brane has completely "disappeared". The process of condensation can also produce lower dimensional unstable branes (or BPS brane — anti-brane pairs in the superstring) as intermediate states.

Reproducing these results in the SFT of [1] is non-trivial. The apparent simplicity of a cubic interaction vertex is deceptive — the condensation involves an infinite number of physical and unphysical scalar fields of arbitarily high mass. Recent progress on the problem involves a level truncation [2], which appears to lead

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to very good agreement with the expected results for some quantities, such as the vacuum energy after condensation. At the same time, it is not clear why and when level truncation works, and it is difficult to study the dynamics of the non-trivial vacuum using this approach.

The worldsheet analysis (see [4] for a recent discussion) suggests that there should exist a choice of coordinates on the space of string fields that is better suited for the study of tachyon condensation. To see that consider, for example, the process in which the tachyon on a D25-brane in the bosonic string condenses to make a lower dimensional Dp-brane with $p < 25$, which is stretched in the directions $(1, 2, \ldots, p)$. This is achieved by considering the path integral on the disk with the worldsheet action

$$\mathcal{S} = \mathcal{S}_0 + \int_0^{2\pi} \frac{d\theta}{2\pi} T(X(\theta))\,, \tag{1.1}$$

where $\mathcal{S}_0$ is the free field action describing open plus closed strings on the disk, θ is an angular coordinate parametrizing the boundary of the disk, and $T(X^{p+1}, \ldots, X^{25})$ is a slowly varying tachyon profile with a quadratic minimum giving mass to the $25 - p$ coordinates transverse to the Dp-brane $\{X^i(\theta)\}$, $i = p + 1, \ldots, 25$. The action (1.1) describes a renormalization group flow from a theory where all 26 $\{X^\mu\}$ satisfy Neumann boundary conditions (corresponding to the 25-brane) to one where the $25 - p$ coordinates $\{X^i\}$ have Dirichlet boundary conditions (the p-brane).

Any profile $T(X)$ with the above properties will do, but a particularly simple choice is

$$T(X) = a + \sum_{i=p+1}^{25} u_i X_i^2\,, \tag{1.2}$$

for which the worldsheet theory is free throughout the RG flow. The parameters a, u_i flow from zero in the UV to infinity in the IR. A crucial point for what follows is that in this flow a, u_i do not mix with any other couplings.

In the spacetime SFT, the p-brane can be constructed as a finite energy soliton [5, 3]. The above worldsheet considerations imply that there must exist a choice of coordinates on the space of string fields in which the tachyon profile is given by (1.2) and no other fields are excited. In the cubic SFT [1] this is not the case — the soliton contains excitations of an infinite number of fields [2]. As we will see below, a more suitable candidate for describing tachyon condensation is the open SFT proposed by Witten in [6] and refined by Shatashvili [7] (see also [8]). We will refer to this string field theory as "boundary string field theory" (B-SFT).

The plan of the paper is the following. In section 2 we briefly review the construction of B-SFT [6]. We comment on the relation of the spacetime action to the boundary entropy of Affleck and Ludwig [9] and to the cubic Chern-Simons string field theory [1].

In section 3 we turn to an example: bosonic open string theory on a D25-brane in flat spacetime. We evaluate the tachyon potential and kinetic (two derivative)

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term and study condensation to the vacuum and to lower branes. The description of the condensation to the vacuum is exact (since the tachyon is the only field that condenses, and its potential is known exactly), while the properties of solitons (corresponding to lower dimensional branes) receive corrections from higher derivative terms in the action, although the two derivative action is in excellent qualitative agreement with the expected exact results.

In section 4 we show that the corrections to the tension of solitons in this SFT can be computed exactly, since to analyze them it is enough to compute the exact action for tachyon profiles of the form (1.2). We use this observation to compute the tensions and show that they are in exact agreement with the expected results. Some comments about the physics of excited open string states and other issues appear in sections 5 and 6. Appendices A and B contain some of the technical details.

A. Gerasimov and S. Shatashvili have independently noticed the relevance of boundary string field theory to the problem of tachyon condensation [10].

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2. A brief review of boundary string field theory

The construction of [6] is aimed at making precise the notion that the configuration space of open string field theory is the space of all two dimensional worldsheet field theories on the disk, which are conformal in the interior of the disk but have arbitrary boundary interactions. Thus, as in (1.1), one studies the worldsheet action

$$\mathcal{S} = \mathcal{S}_0 + \int_0^{2\pi} \frac{d\theta}{2\pi} \mathcal{V} \,, \tag{2.1}$$

where $\mathcal{S}_0$ is a free action defining an open plus closed conformal background, and $\mathcal{V}$ is a general boundary perturbation. We will later discuss the twenty six dimensional bosonic string, for which $\mathcal{V}$ has a derivative expansion (or level expansion) of the form

$$\mathcal{V} = T(X) + A_\mu(X)\partial_\theta X^\mu + B_{\mu\nu}(X)\partial_\theta X^\mu \partial_\theta X^\nu + C_\mu(X)\partial_\theta^2 X^\mu + \cdots \,. \tag{2.2}$$

The boundary conditions on X (in the unperturbed theory) are $\partial_r X^\mu|_{r=1} = 0$. If one wishes to include Chan-Paton indices, the field $\mathcal{V}$ is promoted to an $N \times N$ matrix and the path integral measure on the disk is weighted by

$$e^{-\mathcal{S}_0} \operatorname{tr} P \exp\Big(- \int_0^{2\pi} \frac{d\theta}{2\pi} \mathcal{V} \Big) \,. \tag{2.3}$$

We will mostly restrict to the case $N = 1$ in what follows.

In general, $\mathcal{V}$ is a ghost number zero operator, which nevertheless might depend on the ghosts, and one must also introduce a ghost number one operator $\mathcal{O}$ via

$$\mathcal{V} = b_{-1}\mathcal{O} \,. \tag{2.4}$$

If, as in (2.2), $\mathcal{V}$ is constructed out of matter fields alone, one has

$$\mathcal{O} = c\mathcal{V}\,. \tag{2.5}$$

It is not clear that the theory on the disk described by (2.1) makes sense. Even if one restricts attention to $\mathcal{V}$'s that do not depend on ghosts, such as (2.2), in general the interaction is non-renormalizable and one might expect the theory to be ill-defined. This is an important issue,[1] about which we will have nothing new to say here, however there are clearly interesting cases, such as tachyon condensation, in which the interaction (2.1) is renormalizable. The discussion below definitely applies to these cases and perhaps more generally.

Parametrizing the space of boundary perturbations $\mathcal{V}$ by couplings λ^i:

$$\mathcal{V} = \sum_i \lambda^i \mathcal{V}_i \tag{2.6}$$

(and consequently $\mathcal{O} = \sum_i \lambda^i \mathcal{O}_i$ (2.4)), the spacetime SFT action S is defined by

$$\frac{\partial S}{\partial \lambda^i} = \frac{1}{2}\int_0^{2\pi} \frac{d\theta}{2\pi} \int_0^{2\pi} \frac{d\theta'}{2\pi} \langle \mathcal{O}_i(\theta)\{Q, \mathcal{O}(\theta')\}\rangle_\lambda\,, \tag{2.7}$$

where Q is the BRST charge and the correlator is evaluated with the worldsheet action (2.1). Note that (2.7) defines the action up to an additive constant; also, the normalization of the action is not necessarily the same as in other definitions.[2] We will fix both ambiguities below.

Specializing to $\mathcal{O}$'s of the form (2.5), and using the fact that if $\mathcal{V}_i$ is a conformal primary of dimension Δ_i,

$$\{Q, c\mathcal{V}_i\} = (1-\Delta_i)c\partial_\theta c\mathcal{V}_i \tag{2.8}$$

we conclude from (2.7) that

$$\frac{\partial S}{\partial \lambda^i} = -(1-\Delta_j)\lambda^j G_{ij}(\lambda)\,, \tag{2.9}$$

where

$$G_{ij} = 2\int_0^{2\pi} \frac{d\theta}{2\pi} \int_0^{2\pi} \frac{d\theta'}{2\pi} \sin^2\left(\frac{\theta-\theta'}{2}\right) \langle \mathcal{V}_i(\theta)\mathcal{V}_j(\theta')\rangle_\lambda\,. \tag{2.10}$$

Actually, it is clear that eq. (2.9) cannot be true in general, since it does not transform covariantly under reparametrizations of the space of theories, $\lambda^j \to f^j(\lambda^i)$. Indeed, $\partial_i S$ and G_{ij} transform as tensors (the latter is the metric on the space of worldsheet

[1] which is at the heart of the question of background independence in open SFT, and which was the main motivation for [6].

[2] For example, in the conventional normalization the action goes like $1/g_s$.

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theories), but λ^i does not. The correct covariant generalization of (2.9) was given in [7]. The worldsheet RG defines a natural vector field on the space of theories

$$\frac{d\lambda^i}{d\log|x|} = -\beta^i(\lambda)\,, \tag{2.11}$$

where $|x|$ is a distance scale (e.g. a UV cutoff), and

$$\beta^i(\lambda) = -(1-\Delta_i)\lambda^i + O(\lambda^2) \tag{2.12}$$

is the β function,[3] which transforms as a vector under reparametrizations of λ^i. The covariant form of (2.9) is thus

$$\frac{\partial S}{\partial\lambda^i} = \beta^j G_{ij}(\lambda)\,. \tag{2.13}$$

As is well known in the general theory of the RG, one can choose coordinates on the space of theories such that the β functions are exactly linear.[4] This can always be done locally in the space of couplings, so long as the linear term in the β-function is non-vanishing. In such coordinates, (2.13) reduces to (2.9).

In [6, 7] it was further shown that the action S defined by (2.13) is related to the partition sum on the disk $Z(\lambda^i)$ via

$$S = \left(\beta^i\frac{\partial}{\partial\lambda^i} + 1\right) Z(\lambda)\,. \tag{2.14}$$

Note that (2.14) fixes the additive ambiguity in S by requiring that at fixed points of the boundary RG (at which $\beta^i(\lambda^*) = 0$)

$$S(\lambda^*) = Z(\lambda^*)\,. \tag{2.15}$$

From the worldsheet point of view, the properties (2.13), (2.14) and (2.15) mean that S is a non-conformal generalization of the boundary entropy of [9]. In fact, in any unitary theory satisfying these properties one can prove the "g-theorem" postulated in [9]. Indeed, the scale variation of S is given by the Callan-Symanzik equation

$$\frac{dS}{d\log|x|} = -\beta^i\frac{\partial}{\partial\lambda^i}S = -\beta^i\beta^j G_{ij}\,, \tag{2.16}$$

where we used the fact that S depends on the scale only via its dependence on the running couplings, and equations (2.11), (2.13). In a unitary theory, the metric

[3]We should note that we are using the particle physics conventions here — the β function is negative for relevant perturbations. In some other papers on the subject, e.g. [9], the opposite conventions are used.

[4]In such coordinates, the BRST charge Q is independent of the couplings, and (2.8) holds everywhere.

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G_{ij} (2.10) is positive definite; thus S decreases along RG trajectories. Finally, the property (2.15) implies that at fixed points of the boundary RG, S coincides with the boundary entropy as defined in [9]. Thus, in any unitary theory in which the considerations of [6] do not suffer from UV subtleties (associated with non-renormalizability), the g-theorem of [9] is valid.

As mentioned above, a natural choice of coordinates on the space of string fields is one in which the β-functions are exactly linear. This choice can always be made locally for $\Delta_i \neq 1$. These coordinates become singular as $\Delta_i \to 1$, which in string theory language is the place where the components of the string field (e.g. $T(X)$, A_μ etc. in (2.2)) go on-shell. On the other hand, since the RG flows are straight lines in these coordinates, they are well suited to studying processes which are far off-shell, such as tachyon condensation, since they minimize the mixing between different modes.

In contrast, the cubic SFT parametrization of worldsheet RG is regular close to the mass shell; it appears to be closely related to the coordinates on coupling space implied by the $\epsilon(= 1 - \Delta)$ expansion.[5] These coordinates are useful for studying processes close to the mass shell, such as reproducing perturbative on-shell amplitudes.

This raises the interesting question of how the action S defined above is related to the cubic action of [1]. It seems clear that the cubic SFT must correspond to (2.13) and (2.14) for a particular choice of coordinates on the space of string fields (or worldsheet couplings). The two sets of coordinates are related by a complicated and highly singular transformation (see appendix A for some comments on this transformation). As we will see below, tachyon condensation is simpler in the coordinates (2.9), as one would expect from the above discussion.

3. A first look at tachyon condensation on the D25-brane in the bosonic string

In this section we will study the action S described in the previous section, restricting to the tachyon field. We will keep terms with up to two derivatives and study various features of tachyon condensation using the resulting action, which will turn out to have the form[6]

$$S = T_{25} \int d^{26}x \left[\alpha' e^{-T} \partial_\mu T \partial^\mu T + (T+1)e^{-T} + \cdots \right], \qquad (3.1)$$

where the $\cdots$ stand for terms with more than two derivatives. Before deriving (3.1), we would like to make a few comments on its form.

[5] For a discussion of the ϵ expansion in boundary two dimensional QFT see e.g. [9].
[6] Our conventions are $\eta_{\mu\nu} = \mathrm{diag}(-1, +1, \ldots, +1)$.

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1. The tachyon potential is

$$U(T) = (T+1)e^{-T}\,. \tag{3.2}$$

   This potential is exact, and indeed already appears in [6, 8]. The perturbative vacuum corresponds to $T = 0$, near which $U(T) = 1 - \frac{1}{2}T^2 + \cdots$. The "stable vacuum" to which the tachyon condenses is at $T = +\infty$, where $U(T) \to 0$. One can ask why the tachyon does not instead roll to $T = -\infty$ where $U(T)$ goes to $-\infty$. We will postpone this issue to section 6.

2. T_{25} in (3.1) is the tension of the D25-brane. Indeed, in the perturbative vacuum $T = 0$, $S = T_{25}V$, where V is the volume of spacetime. Note that our tachyon field T is dimensionless.

3. From (3.1) it seems that the mass of the tachyon in the perturbative vacuum is $\alpha' M^2 = -1/2$. Of course, the correct result is $\alpha' M^2 = -1$, but there is no paradox since the higher derivative terms that have been neglected in (3.1) are important in determining this mass. In appendix A we show that the inverse propagator of the tachyon indeed exhibits a simple pole at $\alpha' k^2 = 1$.

4. The action (3.1) is related by a field redefinition

$$\phi = -2e^{-T/2} \tag{3.3}$$

   to an action studied recently in [11] as a toy model of tachyon condensation. These authors found that this model exhibits some remarkable similarities to tachyon condensation in SFT. We now see that it is in fact a two derivative approximation to the exact tachyon action. As we will discuss later, this clarifies the origin of some of the properties found in [11].

The action (3.1) can be determined from the definitions (2.7) and (2.14) as follows. One starts by evaluating the partition sum Z in (2.14) for the tachyon profile (1.2) (with generic $u_i > 0$ and $p = -1$). This should be possible because, as mentioned in section 1, the resulting worldsheet theory is free for all a, u_i. Plugging into (2.14) one then finds that the action $S(a, u_i)$ is given by

$$S(a, u_i) = \left(-a\frac{\partial}{\partial a} + 1 + \sum 2\alpha' u_i - \sum u_i \frac{\partial}{\partial u_i}\right) Z(a, u_i)\,. \tag{3.4}$$

The action (3.1) can then be reconstructed by taking the limit $u_j \to 0$. A simple scaling argument shows that the leading behavior of the action (3.1) evaluated on the profile (1.2) in the limit $u_j \to 0$ comes from the potential term; terms with $2n$ derivatives are down from the leading term by n powers of u_j. Thus, by examining the first two terms in S (3.4) we can uniquely reconstruct the potential and kinetic

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terms in (3.1). At higher orders in derivatives, there are many terms one can write down (most of which vanish on the profile (1.2)) and one needs additional information to determine the full action (see appendix B).

The partition sum $Z(a, u_i)$ has been computed in [6]. The answer can be written as

$$S(a,u) = \left(a + 1 + \sum 2\alpha' u_i - \sum u_i \frac{\partial}{\partial u_i}\right) e^{-a} \prod Z_1(2\alpha' u_i)\,, \tag{3.5}$$

where

$$Z_1(u) = \sqrt{u} e^{\gamma u} \Gamma(u)\,. \tag{3.6}$$

and γ is the Euler number. For small u_i one finds

$$S = (a+14)e^{-a} \prod_{j=1}^{26} \frac{1}{\sqrt{2\alpha' u_j}} + 2\alpha' e^{-a} \left(\sum_{j=1}^{26} u_j\right) \prod_{j=1}^{26} \frac{1}{\sqrt{2\alpha' u_j}} + \cdots\,. \tag{3.7}$$

The first line of (3.7) should be compared to the potential term in (3.1) evaluated on the profile (1.2); the second must be due to the kinetic term. Evaluating the potential energy one finds

$$T_{25}(2\pi\alpha')^{13} e^{-a}(a+14) \prod_{j=1}^{26} \frac{1}{\sqrt{2\alpha' u_j}}\,. \tag{3.8}$$

Comparing to (3.7) we see that

$$T_{25} = 1/(2\pi\alpha')^{13}\,. \tag{3.9}$$

Of course, the fact that this is not the standard form of the tension is due to the freedom of rescaling the action (2.7), (2.14). We can use (3.9) to determine the multiplicative renormalization of S needed to bring it into standard form. Computing the kinetic term in (3.1) and comparing the result to the second line of (3.7) fixes the coefficient of $(\partial_\mu T)^2$ to be the one given in (3.1).

After deriving the action (3.1) we are now ready to proceed to studying tachyon condensation. The first thing that one might worry about is whether it is enough to study the tachyon dynamics, or whether one must include the infinite number of excited states, as in [2]. We will show later that only the zero momentum tachyon condenses in the coordinates on string field space that we are working in, but for now we will assume that and proceed.

Consider first spacetime independent tachyon profiles. The (locally) stable vacuum is at $T = \infty$. The vacuum energy vanishes there. Since the potential (3.2) is exact, this gives a proof of Sen's conjecture [5] that $U_{\text{pert}} - U_{\text{closed}} = T_{25}$ where U_{pert} is the value of the potential in the perturbative open string vacuum and U_{closed} is the value of the potential in the closed string vacuum where the open strings have "condensed" and "disappeared". We have seen above that at a stationary point, U is just the g-function, and moreover that $U_{\text{closed}} = 0$. On the other hand, it is straightforward to identify the tension of D-branes with the g-function [12, 13].

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Note that in our coordinates the stable vacuum is at infinity. This is not in disagreement with other calculations in which it occurs at a finite value of T [2], since field redefinitions change the value of T at the minimum of the potential. In fact, in coordinates on the space of couplings where the β functions in a theory are exactly linear, any infrared fixed points will *always* occur at infinite values of the couplings.

A more invariant question is what is the distance in field space between the perturbative fixed point at $T = 0$, and the stable minimum at $T = \infty$. For this we need to compute the metric on the space of T's. This is easily done either by using (2.13) (with $S = (T+1)\exp(-T)$, $\beta^T = -T$) or by reading off the metric from the kinetic term in (3.1). Either way one finds that the metric on field space is

$$ds^2 = e^{-T}(dT)^2 \,. \tag{3.10}$$

Thus, the distance between $T = 0$ and $T = \infty$ is finite.[7]

Consider next spacetime dependent tachyon profiles, which describe lower dimensional branes as solitons. The equations of motion following from the action (3.1) are

$$2\alpha' \partial_\mu \partial^\mu T - \alpha' \partial_\mu T \partial^\mu T + T = 0 \,. \tag{3.11}$$

We are looking for finite action solutions which asymptote to the "stable vacuum" $T = \infty$. The solutions are in fact precisely the profiles (1.2) that entered our discussion a number of times before! Substituting (1.2) into (3.11) one finds that each of the u_i is either 0 or $1/4\alpha'$. That is, the solitons are translationally invariant along a linear subspace of R^{26} and spherically symmetric transverse to that subspace. Let n be the number of nonzero u_i's. Then $a = -n$. We interpret such codimension n solitons as D$(25 - n)$-branes.

Substituting into the action (3.1) gives

$$S = T_{25} \left(e\sqrt{4\pi\alpha'} \right)^n V_{26-n} \,. \tag{3.12}$$

Comparing this to the expected tension $T_{25-n} V_{26-n}$ we conclude that

$$\frac{T_{25-n}}{T_{25}} = \left(\xi 2\pi \sqrt{\alpha'} \right)^n \,, \tag{3.13}$$

with $\xi = e/\sqrt{\pi} \cong 1.534$. We have written it this way to facilitate comparison with the exact answer

$$\frac{T_{25-n}}{T_{25}} = \left(2\pi \sqrt{\alpha'} \right)^n \,. \tag{3.14}$$

[7]This distance is exactly calculable in our approach, and it would be interesting to compare it to level truncated cubic SFT [2]. This would involve computing the kinetic term for the tachyon and the other fields that condense in level truncated SFT.

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In the next section we will see that one can improve on the result (3.13) and calculate the tensions (3.14) exactly. But before moving on to that analysis it is useful to make a few remarks about the results obtained so far.

One striking feature of the foregoing discussion is the fact that the soliton solutions are given precisely by the quadratic tachyon profiles that play such a prominent role in the worldsheet analysis. This explains why studying them is so easy: the worldsheet theory in their presence remains free! It also makes it clear why we are getting descent relations of the form (3.13): as explained above, the action (3.1) is nothing but the boundary entropy, and for spherically symmetric profiles of the form

$$T(X) = -n + u \sum_{i=1}^{n} X_i^2 \tag{3.15}$$

the boundary entropy factorizes. Finally, it is clear why we are not getting the correct descent relation (3.14) but rather an approximate version (3.13). The reason is that at this level of approximation we find a finite value of the mass parameter u in (3.15). So the action (3.1) is approximately computing the boundary entropy at a finite point along the RG trajectory. Since, as discussed in section 2, the boundary entropy is a monotonically decreasing quantity, we expect to find a larger answer at finite u than at the infrared fixed point ($u = \infty$). This is the reason why the parameter ξ in (3.13) is larger than one.

All this makes it clear that what will happen when we include higher derivative corrections in (3.1) is that the soliton profiles will still have the form (3.15), but $|a|$ and u will increase to infinity. We will demonstrate that this is indeed the case in the next section.

The codimension n solitons (3.15) were also discussed recently in [11]. Our results are in exact agreement with those of [11], although the interpretation is slightly different. The authors of [11] analyzed the spectrum of small fluctuations around the solitons (3.15). They found a discrete spectrum of scalars with masses

$$\alpha' M_n^2 = \frac{1}{2}(n-1)\,; \qquad n = 0, 1, 2, \ldots \tag{3.16}$$

This is very natural from the worldsheet point of view as well, since once we turn on a worldsheet potential of the form (3.15) (even for finite u), it is clear that one expects to find only fields that are bound to the lower dimensional brane but otherwise have the same properties as their higher dimensional cousins.

Finally, one might wonder whether it is possible to describe multi-soliton configurations in the theory (3.1). From the worldsheet point of view this involves [4] studying multicritical tachyon profiles of the form

$$T(X) = \sum_{j=1}^{l} a_j |\vec{X}|^{2j}\,. \tag{3.17}$$

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For $l > 1$ the worldsheet theory is no longer free and one expects complications having to do with the interactions between the solitons (fundamental strings connecting different D-branes). Plugging (3.17) into (3.11) we see that the reflection of this in spacetime is that one needs to keep higher derivative terms in the action to study such configurations.

4. An exact calculation of D-brane tensions

We would like to compute the corrections to the descent relations (3.13) coming from higher derivative corrections to the action (3.1). In principle, one might proceed as follows. First generalize the procedure of section 3 to compute higher derivative corrections to the action, and then use the resulting action to determine the profile of the solitons and their tensions. This looks difficult; computing the higher derivative corrections involves both technical and conceptual complications. Also it is likely that the resulting action would be rather unwieldy and difficult to study.

Some of the technical complications can be seen by looking at the action for quadratic tachyon profiles (3.5) and (3.6). As discussed in section 3, implicit in the action $S(a, u_i)$ is an infinite series of higher derivative corrections to (3.1) which can be computed by expanding S in powers of the u_j. An example of such an infinite-derivative action is given in appendix B.

Unfortunately, (3.5) and (3.6) do not determine the tachyon action uniquely, since it is easy to write an infinite number of terms which annihilate the profile (1.2) and thus do not contribute to $S(a, u_i)$. Nevertheless, the discussion of the previous section makes it clear that there is an alternative way to proceed that circumvents all of the above complications and can be used to compute the tensions of the solitons exactly. The basic observation is that *we know that the exact profile of the soliton in the exact SFT (2.13), (2.14) is going to be of the form (1.2), with some particular values of a, u_i*. The reason is that this mode does not mix with any other modes in the SFT (as will be shown in the next section).

Thus, all we have to do to compute the exact tension of the D-brane solitons is to take the exact action $S(a, u_i)$ given by (3.5), (3.6), and extremize it in a and u_i. Furthermore, we know that the extremum we are looking for is one in which n of the $u_i \to \infty$ and the rest vanish (for a codimension n soliton). We next describe this calculation.

For simplicity let us consider first a codimension one soliton. We would like to substitute the ansatz $T = a + uX_1^2$ in (3.1) (with the other $u_i = 0$) and set the action equal to $V_{24+1}T_{24}$. Of course, the action (3.5), (3.6) diverges when $u_i \to 0$, which is a reflection of the divergent volume V_{24+1}. In order to do the computation in a well defined way we must regularize the volume divergence. We do this by periodic identification of

$$X^\mu \sim X^\mu + R^\mu\,, \qquad \mu = 2, \ldots, 26\,. \tag{4.1}$$

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We must now determine the correct normalization of the path integral Z. The correct normalization for the worldsheet zero-mode of an uncompactified spacetime coordinate X is

$$\int_{-\infty}^{\infty} \frac{dX}{\sqrt{2\pi\alpha'}} e^{-\int_0^{2\pi} \frac{d\theta}{2\pi} T(X(\theta))} . \tag{4.2}$$

We know this because if we substitute $T = a + uX^2$, we reproduce $e^{-a}/\sqrt{2\alpha' u}$. It follows that when we periodically identify X^μ as in (4.1) in directions $\mu = 2, \ldots, 26$ and take $T = a + uX_1^2$ the resulting boundary string field theory action is, exactly,

$$S = \left(a + 1 - u\frac{\partial}{\partial u} + 2\alpha' u\right) e^{-a} Z_1(2\alpha' u) \prod_{\mu=2}^{26} \left(\frac{R^\mu}{\sqrt{2\pi\alpha'}}\right) . \tag{4.3}$$

As discussed above, the dynamical variables in this action are a, u. Therefore, we should minimize S with respect to them. Minimizing first with respect to a we find

$$a_* = a(u) = -2\alpha' u + u\frac{d}{du} \log Z_1(2\alpha' u) . \tag{4.4}$$

Substituting back into the action we get:

$$S = \exp[\Xi(2\alpha' u)] \prod_{\mu=2}^{26} \left(\frac{R^\mu}{\sqrt{2\pi\alpha'}}\right) \tag{4.5}$$

where we define

$$\Xi(z) := z - z\frac{d}{dz} \log Z_1(z) + \log Z_1(z) . \tag{4.6}$$

We may now invoke Witten's result (3.6). The action (4.5) is a monotonically decreasing function of u, and therefore the minimization pushes u to ∞, as expected from the worldsheet renormalization group arguments (the g-theorem).

We are particularly interested in the value of the action at the end of the RG trajectory. From Stirling's formula we find at large z

$$\begin{aligned} \log Z_1(z) &\sim z\log z - z + \gamma z + \log\sqrt{2\pi} + \mathcal{O}\left(\frac{1}{z}\right) , \\ \Xi(z) &\sim \log\sqrt{2\pi} + \frac{1}{(6z)} + \mathcal{O}\left(\frac{1}{z^2}\right) . \end{aligned} \tag{4.7}$$

We thus obtain the boundary string field theory action

$$\sqrt{2\pi} \prod_{\mu=2}^{26} \left(\frac{R^\mu}{\sqrt{2\pi\alpha'}}\right) . \tag{4.8}$$

On the other hand, from the spacetime point of view this is clearly equal to $T_{24} \prod_\mu R^\mu$. We therefore conclude that

$$T_{24} = 2\pi\sqrt{\alpha'} T_{25} , \tag{4.9}$$

which is *precisely* the expected value!

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Clearly this exercise can be repeated for branes of higher codimension. After minimization with respect to a we find the action for the codimension n soliton:

$$\exp\left[\sum_{i=1}^{n}\Xi(2\alpha' u_i)\right]\prod_{\mu=n+1}^{26}\left(\frac{R^\mu}{\sqrt{2\pi\alpha'}}\right) \tag{4.10}$$

and therefore each codimension leads to an extra factor of $2\pi\sqrt{\alpha'}$, in agreement with (3.14).

We finish this section with a few comments:

1. The solitonic solutions describing lower dimensional D-branes constructed in section 3 had a finite size, of order $l_s = \sqrt{\alpha'}$ (since their profiles were given by (3.15) with $u = 1/4\alpha'$). In the exact problem, the sizes of the solitons go to zero like $1/\sqrt{u}$. This is in nice correspondence with the usual description of D-branes as (classically) pointlike objects. In level truncated SFT, the lower D-branes were found to correspond to finite size lumps, similar to those of section 3. Here, we saw that the higher derivative terms in the action play a crucial role in reducing the size of the soliton from l_s to zero. Since in the level truncation scheme the contributions of such terms seem to increase with level, it is possible that if the calculations of [3] were continued to much higher levels, the size of the solitons would slowly decrease to zero, as it does in our approach. Another possibility is that the complicated relation of our parametrization of the space of string fields to that of cubic SFT transforms the δ-function tachyon profile we find to a finite size lump.

2. The fact that we have been able to reproduce exactly the tension ratios (3.14) may at first sight seem puzzling. The full spacetime classical SFT is a very complicated theory, with an infinite number of fields and a rich pattern of non-polynomial interactions. The fact that one can prove that this theory has finite action solitonic solutions with profiles and tensions that can be computed exactly looks from the spacetime point of view like a "string miracle". Such "miracles" are very generic in string theory. The oldest example is perhaps (channel) duality of the tree level S-matrix. The fact that an infinite sum over massive s-channel poles can produce a t-channel pole is due in spacetime to an incredible conspiracy of the masses and couplings of Regge resonances. Describing this in terms of a spacetime lagrangian seems hopeless. However, on the worldsheet, this is one of the many consequences of conformal invariance and is easily described and understood. In the tachyon condensation problem, something very similar happens. The miracle is explained by noting that the spacetime action is nothing but the boundary entropy (see section 2), and the process of condensation is trivial since it corresponds to free field theory on the worldsheet (1.2).

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5. Comments on excited open strings

In our discussion so far we focused on the physics of the tachyon. It is interesting, and for some purposes necessary, to generalize the discussion to include excited open strings.

The first question that we address is one that was noted a few times in the text: why can we study condensation of the tachyon without taking into account other modes of the string? The reason is that we can divide the coordinates on field space into a, u, which are free field perturbations and an orthogonal set of coordinates λ_i corresponding to the non-zero momentum modes of the tachyon and excited open string modes. The λ_i could be e.g. modes of one of the fields A_μ, $B_{\mu\nu}$, C_μ in (2.2). It is consistent to set all the excited string modes λ_i to zero in the presence of a tachyon profile of the form (1.2) if and only if the action (2.13), (2.14) does not have any linear terms in any of the couplings λ_i in the background (1.2).

It should be emphasized that while the couplings λ_i are in general non-renormalizable (since they correspond to irrelevant operators with $\Delta_i > 1$), we are treating the dependence of S on λ_i perturbatively. There is no problem with calculating integrated correlation functions of irrelevant operators in a background such as (2.1), perturbed from a conformal background by relevant and marginal operators, at least after suitable regularization and renormalization procedures are specified. One such procedure is described in appendix A (by contrast, studying a worldsheet action like (2.1) with a finite perturbation by an irrelevant operator is likely to lead to inconsistencies.) Accordingly, we may write the action in the form

$$S(a,u,\lambda^i) = S^{(0)}(a,u) + S^{(1)}_i(a,u)\lambda^i + S^{(2)}_{ij}(a,u)\lambda^i\lambda^j + \cdots , \tag{5.1}$$

where $S^{(0)}(a,u)$ is the action (3.5) and we would like to prove that $S^{(1)}_i(a,u) = 0$. Suppose, on the contrary that $S^{(1)} \neq 0$. Then $\partial_i S|_{\lambda=0} \neq 0$. Looking back at equation (2.13) we see that this means that if the metric G_{ij} is non-degenerate on the space of couplings orthogonal to a, u, then $\beta^j(a,u;\lambda^i = 0) \neq 0$.

Now, after fixing string gauge invariances, the metric G_{ij} is non-degenerate in the background (1.2), which corresponds to free field theory. At the same time, the statement that $\beta^i(\lambda)$ does not vanish at $\lambda^i = 0$ implies that as we turn on a and the u's, the λ^i start flowing according to (2.11). But we know that this is false. In free field theory no new couplings are generated by the RG flow. Therefore, $S^{(1)}_i(a,u)$ must vanish. We conclude that all other string modes appear at least quadratically in the spacetime action in the tachyon backgrounds (1.2), and they can be consistently set to zero when studying tachyon condensation.

Again, it is interesting to contrast the situation with the cubic SFT. In this case a higher string mode, call it schematically v, can couple to the tachyon T schematically as $v^2 + vT^2 + v^2T + v^3$. The couplings of the form vT^2 are generically nonzero, and

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indeed the explicit computations [2, 3] show that higher string modes do obtain nontrivial expectation values during tachyon condensation.

Another interesting circle of questions surrounds the fate of the excited string modes as $T \to \infty$. From the worldsheet analysis it is expected that they all "disappear" from the spectrum, but the precise mechanism by which this happens in spacetime is not well understood. It has been proposed [14] that the coefficients of the kinetic terms vanish at the "stable minimum" but the situation is unclear. The viewpoint of this paper sheds some light on these issues.

We would like to construct the action for excited open string modes using the prescription (2.13) and (2.14). We may determine the dependence on the zero mode of the tachyon as follows. Consider the theory in the background $T = a$ (corresponding to (1.2) with $u_i = 0$). The partition sum has in this case a simple dependence on a,

$$Z(a, \lambda_i) = e^{-a} \tilde{Z}(\lambda_i)\,, \tag{5.2}$$

where we denoted all the other modes collectively by λ_i. The action (2.14) therefore takes the form

$$S(a, \lambda_i) = \left(a + 1 + \beta^i \frac{\partial}{\partial \lambda_i} \right) e^{-a} \tilde{Z}(\lambda_i)\,. \tag{5.3}$$

Recalling the form of the exact tachyon potential (3.2) the action (5.3) can be rewritten as

$$S(a, \lambda_i) = U(T) \tilde{Z}(\lambda_i) + e^{-T} \beta^i \frac{\partial}{\partial \lambda_i} \tilde{Z}(\lambda_i)\,. \tag{5.4}$$

As we show in appendix A, near the mass shell (i.e. as $\Delta_i \to 1$), the quadratic term in the partition sum exhibits a first order zero ($\propto 1 - \Delta_i$); thus the usual kinetic terms for the modes λ_i come from the first term on the r.h.s., while the second term, which goes like $(1 - \Delta_i)^2$ near the mass shell, contributes higher derivative corrections. In any case, we see that all terms in the action go to zero as the tachyon relaxes to $T = \infty$, but, at least in these coordinates on the space of string fields, they do not all go like $U(T)$.

A simple application of (5.4) is to the dependence of the Born-Infeld action on T discussed in [14]. A constant $F_{\mu\nu}$ on the D25-brane does not break conformal invariance, and therefore the second term in (5.4) vanishes in this case. The partition sum in the presence of the constant electro-magnetic field is the Born-Infeld action (for a review see [15]),

$$Z(F) = \mathcal{L}_{BI}(F)\,. \tag{5.5}$$

Substituting into (5.4) we conclude that the action for slowly varying gauge fields and tachyons is

$$S = U(T) \mathcal{L}_{BI} \tag{5.6}$$

in agreement with [14] (essentially the same result already appears in [8].)

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One can also use our construction to study the spectrum of the open string theory in the background of a soliton. This involves computing the partition sum Z to quadratic order in the couplings λ_i in the soliton background (1.2) and should give rise to the standard picture of states bound to the soliton (or lower dimensional brane). As we have mentioned, this should help to explain some results of [11].

6. Many open problems

There is a large number of open problems associated with the circle of ideas explored in this paper. In this section we list a few.

It would be interesting to calculate additional terms in the SFT action. This involves both the determination of higher derivative corrections to (3.1) and the inclusion of excited string modes discussed in the previous section. As noted in the text, the exact action (3.5) implies an infinite number of higher derivative corrections to (3.1), but in order to calculate all terms of a given order in derivatives, more information is needed. Perhaps, additional information can be obtained by solving the worldsheet theory corresponding to the multi-soliton tachyon profiles (3.17).

A related problem is understanding more clearly the relation between boundary string field theory and the cubic SFT. It is conceivable that the space of 2d field theories is a nontrivial infinite dimensional space with no good global coordinate system. It appears from the singular relation between the fields (see e.g. appendix A) that coordinates appropriate to the cubic SFT might have a range of validity which is geodesically incomplete and does not coincide with the "patch" in which good coordinates for boundary SFT are valid.

The discussion throughout this paper has focused on the bosonic string, but the construction of section 2 is more general. In particular, the worldsheet RG picture has been generalized to the superstring [4], where it applies to non-BPS D-branes, D-$\bar{\text{D}}$ systems and related configurations. It would be interesting to generalize the considerations of this paper to these problems, especially because the generalization of the cubic SFT to the superstring is subtle and complicated.

Another interesting problem involves the role of quantum effects in the tachyon condensation process.[8] Our discussion here was entirely classical, and yet we found that the action goes to zero as the tachyon condenses (5.4). Usually this is taken to be a sign of strong coupling, and indeed there were proposals in the literature that tachyon condensation leads to a strongly coupled string theory. For example, the form (5.6) of the gauge field action seems naively to suggest that the effective Yang-Mills coupling behaves as

$$\frac{1}{g_{YM}^2} = \frac{U(T)}{g_s} \tag{6.1}$$

[8] We thanks T. Banks and S. Shenker for a discussion of this issue.

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and therefore, as $U(T) \to 0$ the gauge theory becomes more and more strongly coupled.

On the other hand, the worldsheet analysis of [4] seems to suggest that no strong coupling behavior should be encountered as the tachyon condenses, since diagrams with many holes are not becoming larger in this process.

Boundary SFT seems to lead to the same conclusion. It is natural to expect that quantum corrections to the string field action S (2.14) come from performing the worldsheet path integral over Riemann surfaces with holes. Each hole contributes a factor of g_sN as usual (for N D25-branes), as well as a factor of $\exp(-T)$ from the path integral of (1.1). Thus, it looks like the effective coupling is in fact

$$\lambda = g_s N e^{-T} \tag{6.2}$$

and the perturbative expansion looks like

$$Z = N^2 e^{-2T}\left(\frac{1}{\lambda}A_{-1} + A_0 + \lambda A_1 + \cdots\right), \tag{6.3}$$

where $A_{-\chi}$ is obtained from the path integral on surfaces of Euler character χ and no handles. Eq. (6.3) suggests that the theory in fact remains weakly coupled as $T \to \infty$, but this seems difficult to reconcile with the Feynman diagram expansion arising from the coupling (6.1). It would be interesting to resolve this apparent contradiction.

The behavior of the effective string coupling (6.2) is related to another issue raised earlier in the paper.[9] Recall that the tachyon potential (3.2) is not bounded from below as $T \to -\infty$. Even if the tachyon condenses to the locally stable vacuum at $T = +\infty$ (the closed string vacuum), the system will tunnel through the potential barrier to the true vacuum at $T = -\infty$. The instanton responsible for this tunneling is the Euclidean bounce solution corresponding to a codimension twenty six brane in our construction. Like all the other solitons, it has one negative mode, and therefore mediates vacuum decay. It is natural to ask what is the nature of this instability.

The behavior of the effective coupling (6.2) provides a hint for a possible answer. We see that as the system rolls towards the "true vacuum" at $T \to -\infty$, the string coupling grows. This is significant since as is well known, quantum mechanically, open strings can produce closed strings, and in particular, in this case, the closed string tachyon. Thus, one is led to interpret the instability of the "closed string vacuum" at $T \to \infty$ to decay to the "true vacuum" at $T \to -\infty$ as the closed string tachyon instability. While this is a speculation that needs to be substantiated, we note the following as (weak) evidence for it:

1. The amplitude for false vacuum decay due to the bounce goes like $\exp(-1/g_s)$. The fact that it vanishes to all orders in g_s is consistent with the fact that no

[9] We thank P. Horava and H. Liu for useful comments on this issue.

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such instability is observed in perturbative open string theory [4]. Understanding the precise dependence on g_s probably requires a better understanding of the issues discussed around equation (6.2).

2. The fact that the string coupling grows after closed string tachyon condensation, suggested by (6.2), is consistent with the known physics of closed string tachyon condensation. In this process the central charge of the system decreases, and the dilaton becomes non-trivial (linear in one of the coordinates). This leads to strong coupling somewhere in space.

3. For unstable D-branes in the superstring, the corresponding tachyon potential does not have a similar instability, in accord with the fact that there is no tachyon in the closed string sector in that case.

Acknowledgments

We would like to thank T. Banks, M. Douglas, J. Harvey, E. Martinec, N. Seiberg, S. Shenker, and A. Strominger for useful discussions, and J. Harvey for comments on the manuscript. We also thank the participants of the Rutgers group meeting for many lively questions during a presentation of these results. The work of D.K. is supported in part by DOE grant #DE-FG02-90ER40560. The work of G.M. and M.M. is supported by DOE grant DE-FG02-96ER40949. D.K. thanks the Rutgers High Energy Theory group for hospitality during the course of this work. M.M. would like to thank the High Energy Theory group at Harvard for hospitality in the final stages of this work.

A. Some technical results

The string field theory action (2.7) for a tachyon profile

$$T(X) = \int dk\, \phi(k)\, \exp(ik \cdot X) \tag{A.1}$$

is an infinite series $S^{(2)} + S^{(3)} + S^{(4)} + \cdots$ in powers of T. In this appendix we give explicit formulae for the first two terms in this expansion. More generally, we will show that the quadratic term in the string field action for a primary field $\mathcal{V}_i$ has a pole at $\Delta_i = 1$.

The structure of the quadratic term $S^{(2)}$ for a primary field $\mathcal{V}_i$ has a rather simple expression. We only need the correlation function of the boundary operator in the free field theory, which is given by

$$\langle \mathcal{V}_i(\theta)\mathcal{V}_i(\theta')\rangle = \frac{c}{\left[\sin^2\left(\frac{\theta-\theta'}{2}\right)\right]^{\Delta_i}}, \tag{A.2}$$

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and c is a constant. We also need the value of the following integral:

$$\int_0^{2\pi}\frac{d\theta}{2\pi}\int_0^{2\pi}\frac{d\theta'}{2\pi}\left[\sin^2\left(\frac{\theta-\theta'}{2}\right)\right]^z = 4^{-z}\frac{\Gamma(1+2z)}{\Gamma^2(1+z)} = \frac{1}{\pi^{1/2}}\frac{\Gamma(1/2+z)}{\Gamma(1+z)}\,. \tag{A.3}$$

Notice that in the evaluation of this integral we have regulated the short-distance singularities by analytic continuation in z. We will now compute $S^{(2)}$ using (2.14). The term of order $(\lambda^i)^2$ in the partition function is given by:

$$Z^{(2)} = \frac{c}{2}(\lambda^i)^2\int_0^{2\pi}\frac{d\theta}{2\pi}\int_0^{2\pi}\frac{d\theta'}{2\pi}\left[\sin^2\left(\frac{\theta-\theta'}{2}\right)\right]^{-\Delta_i} = \frac{c(\lambda^i)^2}{\pi^{1/2}}\frac{\Gamma(3/2-\Delta_i)}{(1-2\Delta_i)\Gamma(1-\Delta_i)}\,. \tag{A.4}$$

We see that this gives a simple pole in the propagator at $\Delta_i = 1$, a fact that was used in section 5. The action S (2.14) to quadratic order is then:

$$S^{(2)} = (1-2(1-\Delta_i))Z^{(2)} = -\frac{c(\lambda^i)^2}{\pi^{1/2}}\frac{\Gamma(3/2-\Delta_i)}{\Gamma(1-\Delta_i)}\,. \tag{A.5}$$

Notice that the term $\beta_i\partial^i Z$ gives a second order pole in the propagator at this order, as stated in section 5. It is easy to check that the definition (2.13) gives the same answer for $S^{(2)}$.

In the case of the tachyon field (A.1), the correlation function in free field theory is

$$\langle \mathrm{e}^{ik\cdot X(\theta)}\mathrm{e}^{ik'\cdot X(\theta')}\rangle = (2\pi)^d\delta(k+k')\left[\sin^2\left(\frac{\theta-\theta'}{2}\right)\right]^{-\alpha' k^2}, \tag{A.6}$$

and the quadratic piece of the action (A.5) reads in this case

$$S^{(2)} = -\int dk\,\frac{1}{\pi^{1/2}}\frac{\Gamma(3/2-\alpha' k^2)}{\Gamma(1-\alpha' k^2)}(2\pi)^d\phi(k)\phi(-k)\,. \tag{A.7}$$

Notice again that the propagator, which is a complicated function of k^2, exhibits the required pole at $\alpha' k^2 = 1$.

The cubic term for the tachyon field (A.1) can be computed by evaluating the correlation function (A.6) at next order in perturbation theory. The result is:

$$\begin{aligned} S^{(3)} = -\frac{1}{3!}\int & dk dk' dk''\,\phi(k)\phi(k')\phi(k'')(2\pi)^d\delta(k+k'+k'')4^{-(a+b+c)+1}(1-\alpha' k'^2)\times \\ &\times\frac{\Gamma(1+2a)\Gamma(1+2b)}{\Gamma^2(1+a-c)\Gamma^2(1+b-c)}{}_3F_2(-2c,-a-c,-c-b;1+a-c,1+b-c;1)\,, \end{aligned} \tag{A.8}$$

where

$$\begin{aligned} a &= -\alpha' k\cdot k' + 1 \\ b &= -\alpha' k\cdot k'' \\ c &= -\alpha' k'\cdot k''\,, \end{aligned} \tag{A.9}$$

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and the hypergeometric function ${}_pF_q$ is defined by

$$ {}_pF_q(\alpha_1,\ldots,\alpha_p;\beta_1,\ldots,\beta_q;z) = \sum_{n=0}^{\infty}\prod_{i=1}^{p}\left(\frac{\Gamma(\alpha_i+n)}{\Gamma(\alpha_i)}\right)\prod_{j=1}^{q}\left(\frac{\Gamma(\beta_j)}{\Gamma(\beta_j+n)}\right)\frac{z^n}{n!}\,. \tag{A.10}$$

We can now try to compare the action $S = S^{(2)} + S^{(3)} + \cdots$ to the cubic action obtained in the open string field theory of [1]. Using, for example, the approach of [16], one finds

$$\begin{aligned} S^{CS} = {} & A\int dk\,(2\pi)^d\widehat{\phi}(k)\widehat{\phi}(-k)(\alpha' k^2-1) + \\ & + \int dk dk' dk''\, B(k,k',k'')\widehat{\phi}(k)\widehat{\phi}(k')\widehat{\phi}(k'')(2\pi)^d\delta(k+k'+k'')\,, \end{aligned} \tag{A.11}$$

where

$$A = -\frac{1}{2g_s^2}\,,\qquad B(k,k',k'') = -\frac{1}{3g_s^2}\left(\frac{4}{3\sqrt{3}}\right)^{-a-b-c-2}. \tag{A.12}$$

If we assume that the tachyon fields $\phi(k)$ and $\widehat{\phi}(k)$ are related as follows,

$$\widehat{\phi}(k) = f_1(k)\phi(k) + \int dk' f_2(k,k')\phi(k')\phi(k-k') + \cdots\,, \tag{A.13}$$

in such a way that

$$S^{CS}(\widehat{\phi}(k)) = \kappa\, S(\phi(k))\,, \tag{A.14}$$

where κ is a nonzero constant, we obtain:

$$(f_1(k^2))^2 = \frac{\kappa}{\pi^{1/2}A}\frac{\Gamma(3/2-\alpha' k^2)}{\Gamma(2-\alpha' k^2)}\,, \tag{A.15}$$

where we have used that $f_1(k) = f_1(-k)$ (this follows from reality of the tachyon field). By comparing the cubic terms, we find:

$$\begin{aligned} A(\alpha' k^2-1)\left(f_1(k)f_2(-k,k'') + f_1(k')f_2(-k',k'')\right) + B(k,k',k'')f_1(k)f_1(k')f_1(k'') = \\ = \frac{\kappa}{3!}4^{-(a+b+c)+1}(\alpha'(k')^2-1)G(k,k',k'')\,, \end{aligned} \tag{A.16}$$

where we have defined

$$G(k,k',k'') = \frac{\Gamma(1+2a)\Gamma(1+2b)}{\Gamma^2(1+a-c)\Gamma^2(1+b-c)}\,{}_3F_2(-2c,-a-c,-c-b;1+a-c,1+b-c;1)\,. \tag{A.17}$$

Notice that $f_1(k)$ is regular and different from zero when the tachyon is on-shell. On the other hand, if we evaluate the relation (A.16) when the three tachyon fields are on-shell, we find that $G(k,k',k'') = 0$, and therefore $f_2(k,k'')$ must have a pole with nonzero residue at $\alpha' k^2 = 1$. This shows that the relation between the CS and the B-SFT tachyon fields becomes singular on-shell.

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B. Some higher derivative terms in the tachyon action

In this appendix we give an example of a higher derivative lagrangian for the tachyon which reproduces the exact action $S(a,u)$. This is simply meant to indicate the nature of some of the terms. We stress at the outset that the following does not determine an infinite set of couplings, namely, anything which vanishes on the gaussian profile. One unambiguous conclusion one can draw from this exercise is that in terms of $\phi \sim e^{-T}$ the higher derivative terms must be singular at $\phi = 0$.

It is useful to generalize the tachyon profile to $T = a + u_{\mu\nu}X^\mu X^\nu$ with $u_{\mu\nu}$ positive definite. The exact action may be written as

$$S = \left[a + 1 + 2\alpha' \operatorname{tr}(u) - \operatorname{tr}\left(u\frac{\partial}{\partial u}\right)\right] \frac{e^{-a}}{\sqrt{\det u}} \exp\left[\sum_{k=2}^{\infty} \frac{(-1)^k \zeta(k)}{k} \operatorname{tr} u^k\right] \tag{B.1}$$

(there is a regularization dependent term $\sim \operatorname{tr} u$ in the exponential. With the normal ordering prescription of [6] this term vanishes). Expanding the exponential we obtain a series

$$\sum_{n_k \geq 0} A_{n_k} \prod_k (\operatorname{tr} u^k)^{n_k} = 1 + \frac{1}{2}\zeta(2)\operatorname{tr}(u^2) + \cdots \tag{B.2}$$

where at a given order in scaling under $u \to \alpha u$ the sum is a Schur polynomial.

Now the action becomes:

$$ae^{-a}\frac{1}{\sqrt{\det u}} \sum_{n_k} A_{n_k} \prod_k (\operatorname{tr} u^k)^{n_k} + e^{-a}\frac{1}{\sqrt{\det u}} \sum_{n_k} A_{n_k}(14 - L_0(n_k)) \prod_k (\operatorname{tr} u^k)^{n_k} + \\ +2\alpha'(\operatorname{tr} u)e^{-a}\frac{1}{\sqrt{\det u}} \sum_{n_k} A_{n_k} \prod_k (\operatorname{tr} u^k)^{n_k} \tag{B.3}$$

where it is convenient to define $L_0(n_k) = \sum_k k n_k$.

One straightforward way to reproduce this from a lagrangian proceeds by starting with the ae^{-a} term. This is reproduced by

$$T_{25}\pi^{-13} \int dx T e^{-T} \sum_{n_k} \left(\frac{1}{2}\right)^{L_0(n_k)} A_{n_k} \prod_k \left[(\partial_{\mu_1\mu_2}T)(\partial_{\mu_2\mu_3}T)\cdots(\partial_{\mu_k\mu_1}T)\right]^{n_k} \tag{B.4}$$

In order to account for the second line in (B.3) we add terms

$$T_{25}\pi^{-13} \int dx e^{-T} \sum_{n_k} \left(\frac{1}{2}\right)^{L_0(n_k)} B_{n_k} \prod_k \left[(\partial_{\mu_1\mu_2}T)(\partial_{\mu_2\mu_3}T)\cdots(\partial_{\mu_k\mu_1}T)\right]^{n_k} \tag{B.5}$$

with $B_{n_k} = (1 - L_0(n_k))A_{n_k}$. Finally to get the last line of (B.3) we take

$$\alpha' T_{25}\pi^{-13} \int dx e^{-T} (\partial_\mu T)(\partial_\mu T) \sum_{n_k} \left(\frac{1}{2}\right)^{L_0(n_k)} A_{n_k} \prod_k \left[(\partial_{\mu_1\mu_2}T)(\partial_{\mu_2\mu_3}T)\cdots(\partial_{\mu_k\mu_1}T)\right]^{n_k}. \tag{B.6}$$

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References

[1] E. Witten, *Noncommutative geometry and string field theory*, *Nucl. Phys.* **B 268** (1986) 253.

[2] V.A. Kostelecky and S. Samuel, *The static tachyon potential in the open bosonic string theory*, *Phys. Lett.* **B 207** (1988) 169; *On a nonperturbative vacuum for the open bosonic string*, *Nucl. Phys.* **B 336** (1990) 263;
A. Sen and B. Zwiebach, *Tachyon condensation in string field theory*, *J. High Energy Phys.* **03** (2000) 002 [hep-th/9912249];
N. Moeller and W. Taylor, *Level truncation and the tachyon in open bosonic string field theory*, *Nucl. Phys.* **B 583** (2000) 105 [hep-th/0002237].

[3] J.A. Harvey and P. Kraus, *D-branes as unstable lumps in bosonic open string field theory*, *J. High Energy Phys.* **04** (2000) 012 [hep-th/0002117];
R. de Mello Koch, A. Jevicki, M. Mihailescu and R. Tatar, *Lumps and p-branes in open string field theory*, *Phys. Lett.* **B 482** (2000) 249 [hep-th/0003031];
N. Moeller, A. Sen and B. Zwiebach, *D-branes as tachyon lumps in string field theory*, *J. High Energy Phys.* **08** (2000) 039 [hep-th/0005036].

[4] J.A. Harvey, D. Kutasov and E.J. Martinec, *On the relevance of tachyons*, hep-th/0003101.

[5] A. Sen, *Descent relations among bosonic D-branes*, *Int. J. Mod. Phys.* **A 14** (1999) 4061 [hep-th/9902105].

[6] E. Witten, *On background independent open string field theory*, *Phys. Rev.* **D 46** (1992) 5467 [hep-th/9208027]; *Some computations in background independent off-shell string theory*, *Phys. Rev.* **D 47** (1993) 3405 [hep-th/9210065].

[7] S.L. Shatashvili, *Comment on the background independent open string theory*, *Phys. Lett.* **B 311** (1993) 83 [hep-th/9303143]; *On the problems with background independence in string theory*, hep-th/9311177.

[8] K. Li and E. Witten, *Role of short distance behavior in off-shell open string field theory*, *Phys. Rev.* **D 48** (1993) 853 [hep-th/9303067].

[9] I. Affleck and A.W. Ludwig, *Universal noninteger "ground-state degeneracy" in critical quantum systems*, *Phys. Rev. Lett.* **67** (1991) 161;
Exact conformal field theory results on the multichannel Kondo effect: single fermion Green's function, self-energy, and resistivity, *Phys. Rev.* **B 48** (1993) 7297.

[10] A.A. Gerasimov and S.L. Shatashvili, *On exact tachyon potential in open string field theory*, *J. High Energy Phys.* **10** (2000) 034 [hep-th/0009103].

[11] J.A. Minahan and B. Zwiebach, *Field theory models for tachyon and gauge field string dynamics*, *J. High Energy Phys.* **09** (2000) 029 [hep-th/0008231].

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[12] S. Elitzur, E. Rabinovici and G. Sarkisian, *On least action D-branes*, *Nucl. Phys.* **B 541** (1999) 246 [hep-th/9807161].

[13] J.A. Harvey, S. Kachru, G. Moore and E. Silverstein, *Tension is dimension*, *J. High Energy Phys.* **03** (2000) 001 [hep-th/9909072].

[14] A. Sen, *Supersymmetric world-volume action for non-BPS D-branes*, *J. High Energy Phys.* **10** (1999) 008 [hep-th/9909062].

[15] A.A. Tseytlin, *Born-Infeld action, supersymmetry and string theory*, hep-th/9908105.

[16] A. LeClair, M.E. Peskin and C.R. Preitschopf, *String field theory on the conformal plane, 1. Kinematical principles*, *Nucl. Phys.* **B 317** (1989) 411.

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